# **UNCLASSIFIED** AD NUMBER AD012828 LIMITATION CHANGES TO: Approved for public release; distribution is unlimited. FROM: Distribution authorized to DoD only; Administrative/Operational Use; 05 MAY 1953. Other requests shall be referred to Office of Naval Research, ATTN: Code 280, 875 North Randolph Street, Arlington, VA 22203. Pre-dates formal DoD distribution statements. Treat as DoD only. **AUTHORITY** ONR ltr dtd 9 Nov 1977

Office of Naval Research

Contract N5ori-76

Tasks Order No. 1

NR-078-011

Technical Report

on

Mode Control and Operating Voltages

Of Interdigital Magnetrons

by

Amarjit Singh

May 5, 1953

The research reported in this document was made possible through support extended Cruft Laboratory, Harvard University, jointly by the Navy Department (Office of Naval Research), the Signal Corps of the U. S. Army, and the U. S. Air Force under ONR Contract N50ri-76, T. O. 1.

Technical Report No. 179

Cruft Labo-atory

Harvard University

Cambridge, Massachusetts

## MODE CONTROL AND OPERATING VOLTAGES

OF

### INTERDIGITAL MAGNETRONS

by

Amarjit Singh\*

# Summary

A report is given of experiments relating to resonance frequencies and operating voltages of interdigital magnetrons. Data revealing the dependence of resonance frequencies on resonator parameters are reported. It is shown that an attempt to single out one mode for operation, by short-circuiting certain fingers to the opposite face of the cavity, leads to an unsatisfactory mode spectrum. On the other hand, placing radial vanes in the cavity enables degenerate pairs to be separated and all the modes to be accurately controlled. Results obtained by using this method in operating tubes are given. Experiments with operating tubes show that under a given set of conditions, the first-and second-order modes can each operate at more than one voltage. The different voltages can be attributed to the excitation of different Fourier components of the field configuration

## I. Introduction

In the interdigital magnetron the anode segments consist of fingers joined alternately to the opposite faces of a cylindrical cavity. When the cavity resonates, the field pattern produced in the space between the fingers and the cathode resembles that of the  $\pi$ -mode of a multicavity magnetron. The field in the cavity is either independent of the asimuthal coordinate  $\phi$ , or goes-through one or more complete cycles as  $\phi$  varies from O to  $2\pi$ . The corresponding modes are referred to as the seroth-order mode, first-order mode, etc. The modes of various orders are well separated from one another in frequency.

Formerly of Cruft Laboratory, Harvard University. Now at National Physical Laboratory, New Delhi, India.

However, modes of all orders except the zeroth (hereafter referred to as the higher-order modes) occur in degenerate pairs. Two modes of the same order differ from each other only in the location of the nodes of E-field in the cavity. Normally their frequencies are separated by a small amount due to the presence of asymmetries in the resonator. If operation is desired in one of the higher-order modes, it is necessary to separate the two modes of that order by a sufficiently large interval. If this is not done, stable operation would not be obtained in either mode.

In the work reported here, one objective was to control the frequencies of all the modes up to the second order. Another one was to study the electronic operation of first-and second-order modes.

In the higher-order modes, the field configuration does not exactly correspond to the \$\pi\$-mode, since the E-field in the cavity changes sign at certain locations. In order to make the "polarity of the fingers alternate regularly all the way around the anode," Crawford and Hare used the "phase-reversing anode." At each position of zero E-field, two adjacent fingers were joined to the same side of the cavity. The two could be made as one broad finger, called a phase-shifting finger. Thus the order of joining fingers to the cavity faces was reversed at the places where the E-field changes sign. This was expected to ensure operation in only one mode, at one voltage. However it was found by the author that in spite of the presence of phase-shifting fingers, operation is possible in more than one mode; and for each mode at more than one voltage.

### II. Mode Control

The first step in the design of an interdigital resonator is the choice of dimensions of the anode and the cavity, in such a way that the resonance frequencies lie in the desired range, and the modes of various orders are separated by the desired intervals. The second step is the introduction of asymmetries into the structure, so that the two modes corresponding to each higher order are well separated from each other. In connection with the first step, the modes of various symmetrical interdigital resonators were studied. For the second step the effects of two kinds of asymmetries were studied.

For the sake of simplicity, the dummy cathode was left out in the first place. However, it was observed that the insertion of the cathode lowered the frequency of the zeroth-order mode by about 20 per cent and increased the frequencies of the first-and second-order modes by about 5 per cent. In the case of the zeroth-order mode, the change depended also upon the structure into which the cathode lead entered. This is because the zeroth-order mode couples strongly to the cathode lead.

## Modes of a Symmetrical Interdigital Resonator

The variation of resonance frequency of modes of different orders with changes in resonator parameters, was studied with the help of a demountable resonator. Figure 1 shows a cross section of the resonator, the lower set of fingers being omitted for the sake of clarity. The cavity was formed by clamping two face plates F, F against a thick annular disc D. Fingers attached to two cylinders were inserted through holes in the face plates. The squirrel cage formed by the fingers was made coaxial with the annular disc by fitting the face plates and the disc into an outer cylinder C. Two caps completed the resonator. The parameters of the cavity could be varied by using different annular discs D. The parameters of the anode could be varied by inserting different sets of fingers through the holes in the face plates. Thus several combinations of anode and cavity parameters were easily obtainable. The resonance frequencies were determined by inserting two coupling loops into the cavity, one for seeding in power from a test oscillator, and the other for detecting the amplitude of oscillations. The modes were identified by plotting the field patterns with the help of a rotating probe. The rectified and amplified output of the probe was fed to a recording milliammeter, while the probe was slowly rotated by a motor having a step down gearbox. The field patterns were thus accurately and directly recorded.

Graphs of resonance frequency of the different modes, as a function of resonator parameters, are given in Figs. 2 and 3. There, N stands for the total number of fingers and n stands for the order of the mode. The symbols for other parameters are explained in Fig. 1. The following general conclusions can be drawn from the data:

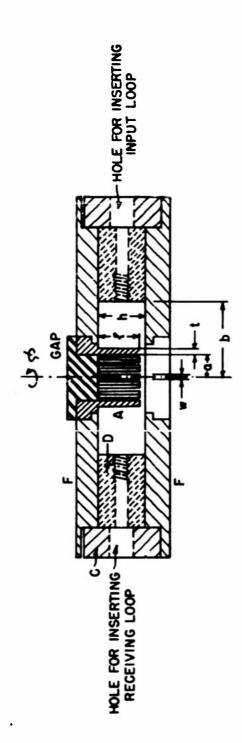
1

2

- a. There is no general correlation between the wavelengths and the length of the "folded transmission line" formed by the fingers, as suggested by Crawford and Hare. 1
- b. Increase of cavity radius reduces the resonance frequency and increases the separation of the different modes.
- c. The ratio by which the resonance frequency changes for a given variation of cavity radius is smaller for the higher-order modes.
- d. Increase of radial thickness of the fingers, or decrease of the separation between fingers, reduces the resonance frequency, the ratio being larger for larger cavity radii. It is also seen that a mode separation of the order of 40 per cent is obtained by choosing the ratio of cavity radius to anode radius to be 3:1.

# Modes of Interdigital Resonator with Shorting Wires at Fingers

In the early stages of this work shorting wires were used to solve the problem of degeneracy in the higher-order modes. At that time the primary interest was in getting only one mode to give steady operation. The secondorder mode was chosen in preference to the zeroth-order mode, because of its higher frequency and because it does not require cathode decoupling chokes for proper operation, as the latter does. The resonator has 24 ordinary fingers, and four phase-shifting fingers located at intervals of 90 degrees. The tips of these four fingers were short-circuited to the opposite face of the cavity. It was expected that all the modes except the second-order mode with nodes of E-field at the shorting wires, would become inoperable. In actual practice the tube could operate at four frequencies, three of which were separated by intervals of only 4 per cent and 2 per cent respectively. The The frequencies for a typical tube were 7190, 5180, 4970, and 4880 Mc/s, respectively. The mode at 5180 Mc/s. gave very much larger power output than any of the others. It was believed to be the mode in which operation was intended. The presence of the other two modes very near to it was thus a serious flaw in the frequency spectrum. In order to remove this flaw, various changes were tried out in the auxiliary structures, such as changes in the cathode lead, output leads etc. The working hypothesis in these earlier



日外の多人は

FIG.1 CROSS SECTION OF DEMOUNTABLE RESONATOR

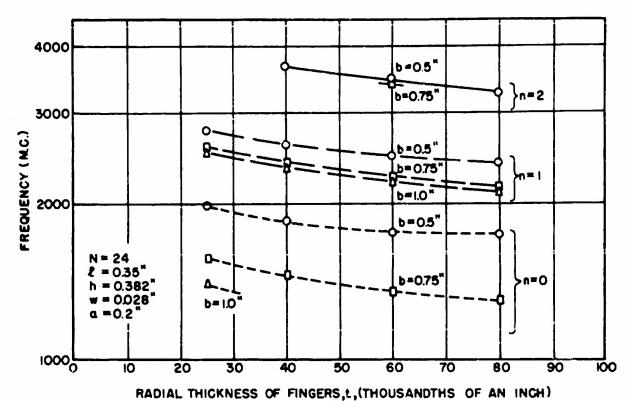


FIG. 2 VARIATION OF FREQUENCY WITH "CAPACITY" AT THE ANODE

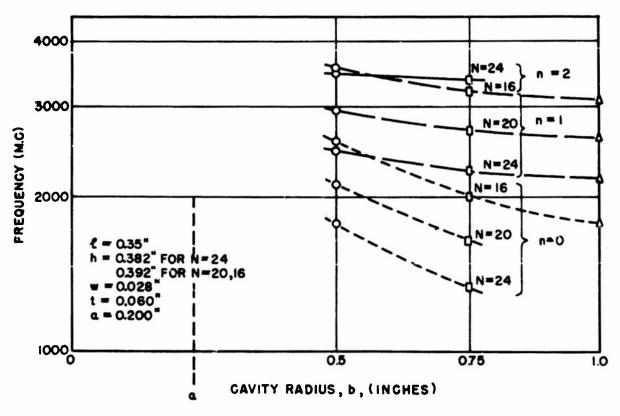


FIG. 3 VARIATION OF FREQUENCY WITH CAVITY RADIUS

attempts was that the three modes close together, were modifications of the second-order mode, caused by resonances in the auxiliary structures. However, the frequency spectrum as revealed by cold tests remained practically unchanged with changes in these structures. Light was thrown on the problem after the technique of plotting field patterns had been refined sufficiently. The patterns shown in Fig. 4 were obtained for the modes at 5180, 4970, and 4880 Mc/s. In this figure, the locations of phase-shifting fingers are denoted by P, and the location of the coupling loop is denoted by L. The patterns revealed that the mode at 5180 Mc/s was a second-order mode with nodes of E-field at the shorting wires. The field pattern of the mode at 4970 Mc/s showed nearly zero E-field in two opposite quadrants. The mode at 4880 Mc/s showed nearly zero E-field in the second pair of quadrants. This led to the conclusion that these latter two modes were first-order modes. They were highly distorted by the presence of shorting wires, which had raised their frequencies to values very close to the undisturbed second-order mode.

The shorting wires were cut, and for the first time the frequency spectrum showed a marked change. The mode at 5180 Mc/s was undisturbed, as expected. The modes at 4970 and 4880 Mc/s disappeared, and new ones appeared at 3920 Mc/s and 3230 Mc/s. These were identified from their field patterns as being, respectively, the first-order and the zeroth-order modes.

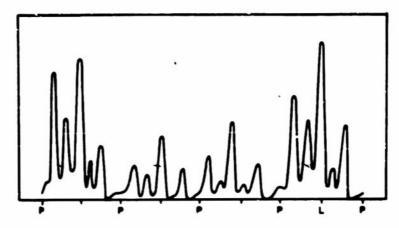
As a further test for the hypothesis that shorting wires had raised the frequencies of the zeroth-and first-order modes, the following experiment was performed. Four shorting wires were introduced between the two face plates, all being at the same distance behind the phase-shifting fingers. It was found that the second-order mode split into two parts, one remaining at the original frequency, and the other being at a higher frequency. The frequencies of the zeroth and first-order modes were found to have risen. This rise could be continuously traced to the vicinity of the undisturbed second-order mode, by putting the shorting wires at smaller and smaller distances behind the fingers. Meanwhile, the higher second-order mode went beyond the frequency range of the oscillator used for cold tests. It was presumed to reach the value 7190 Mc/s when the shorting wires were put on the fingers themselves.

# Modes of Interdigital Resonator with Vanes in the Cavity

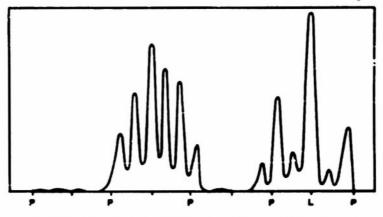
The experiments just described suggested the use of radial vanes in the cavity for the purpose of solving the problem of degeneracy in higher-order modes. Cold tests were performed with a resonator made for an operating tube of the kind described in the last section. Four radial slots were cut in one of the face plates, so as to lie behind the four phase-shifting fingers. Vanes were inserted into these slots and clamped in place. The radial penetrations of the vanes could be varied independently. However, radially opposite vanes were always set at symmetrical locations. Figure 5 shows a cross section of the resonator. The resonance frequencies were determined for all the combinations of a set of values of da and db, where da was the radial length of one pair of vanes and db was that of the other pair.

The results are shown in Fig. 6. It gives graphs of resonance frequencies versus da with various values of db as parameter. It is seen that the frequencies of the zeroth-order mode and of the second-order mode, which ordinarily has maxima of E-field at the position of the vanes, are increased by an increase of da as well as of db. The frequency of each of the first-order modes is independent of one pair of vanes, and rises with increase in penetration of the other pair. The second-order mode, having zeros of E-field at the vanes, is practically unaffected by both da and db. In general, insertion of vanes leaves the resonance frequency of a mode unaltered if the E-field for the undisturbed mode is zero at the positions of the vanes; the resonance frequency is increased when the above condition is not satisfied; the rate of increase of frequency with increase of d becomes larger as d increases.

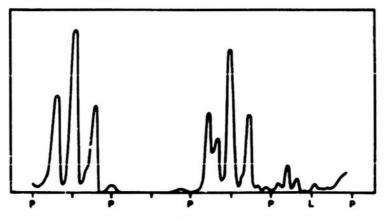
The graphs also show how five different modes can be located at convenient intervals, with the help of two pairs of vanes in the cavity. The five modes are: the zeroth-order mode, the two first-order modes, and the two second-order modes. By adjusting the difference between d and db the separation between the first-order modes can be adjusted. By adjusting the actual magnitudes of da and db the separation between the two second-order modes can be adjusted. If the various intervals finally obtained are not large enough, then the basic intervals between the zeroth-, first-, and second-order modes must be increased by increasing the ratio of cavity radius to anode radius.



a) LOWER SECOND ORDER MODE (5180 M.C.)

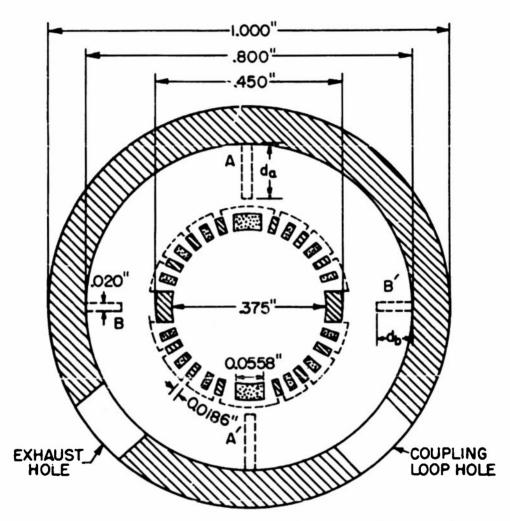


b) HIGHER FIRST ORDER MODE (4970 M.C.)



c) LOWER FIRST ORDER MODE (4880 M.C.)

FIG. 4 FIELD PATTERNS WITH SHORTING WIRES



LENGTH OF FINGERS =.220"
DISTANCE BETWEEN POLE PIECES =0.480"

FIG. 5 CROSS SECTION OF RESONATOR

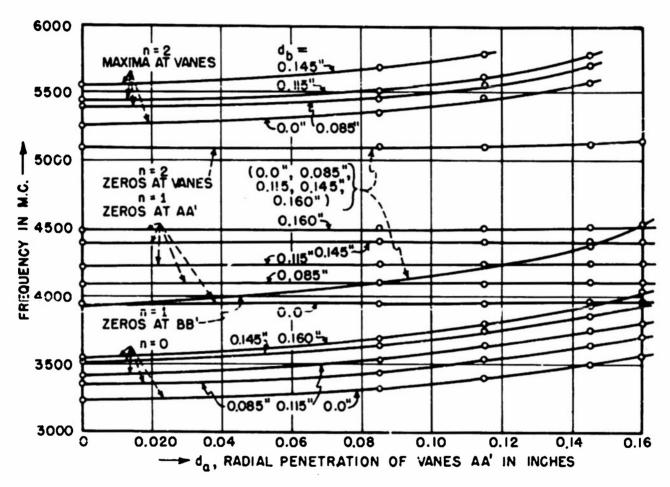


FIG.6 VARIATION OF RESONANCE FREQUENCIES WITH PENETRATION OF VANES

The field patterns of the two second-order and two first-order modes obtained when vanes are used, are shown in Fig. 7. The distortion in patterns is much less than when shorting wires were used. Since phase-shifting fingers are present, a regular field configuration is obtained only with the lower second-order mode.

# Operating Tubes with Vanes in the Cavity

Figure 8 shows a cutaway view of an operating tube, using vanes for mode control. One of the pole pieces and a part of the cavity wall have been removed to show the inner structure. The cathode, the fingers, the cavity, the vanes, the coupling loop, one pole piece, and the exhaust tube can be seen in the picture. The anode was prepared by hobbing. The cathode was of the screen-sleeve type. The dimensions of the fingers and cavity are shown in Fig. 5. The dimensions of the vanes were chosen on the basis of Fig. 6 and are given in Table I.

Table I.

Tube Nc.	Dimensions of		Cathode Diameter	Frequencies of Modes			Mc/s
	Vanes (i	inches)	(Inches)	Second		_	Lower First Order
24		g wires s used.	0, 234	7190	5180	4970	4880
26	0.150	0.073	0, 234	6570	5210	4930	4370
27) 28)	0.138	0.073	0.234	6120	5 210	4780	4370
29	0.138	0.073	0, 272		5290	4840	4430

This tables gives data on frequencies obtained with four tubes using vanes. Data on a tube using shorting wires have been included for comparison. The final intervals obtained with vanes are of the order of 9 per cent compared with the intervals of 4 per cent and 2 per cent given by shorting wires. The interval cotained with vanes was restricted by the fact that the ratio of cavity radius to anode radius of the resonator was only 2:1. Figure 3 shows that this

ratio would give a comparatively small basic separation between the modes of various orders.

## III. Operating Voltage of Higher-Order Modes

As the anode voltage applied to an operating tube was increased, keeping the magnetic field constant, the current varied in a manner shown by Fig. 9. It is seen that operation in the lower second-order mode and lower first-order mode is obtained in two distinct ranges of voltage in each case. The same has been found to be true for the higher first-order mode also, in other cases. Operation in more than two ranges of voltage for a given mode has been observed in some cases. Such observations have been made with tubes having shorting wires as well as with those having vanes. It is clear that even in the lower second-order mode more than one Fourier component can be excited, in spite of the presence of phase-shifting fingers. Thus it is of interest to study the Fourier components of the field configurations of different modes, with and without phase-shifting fingers.

# Fourier Components of the Field Configurations

The azimuthal component  $E_{\phi}(a, \phi)$  of the E-field at the anode, in the median plane can, in the general case, be written as:

$$E_d(a, \phi) = Z(\phi)\cos n\phi P(\phi)$$
, or  $Z(\phi)\sin n\phi P(\phi)$ ,

where  $Z(\phi)$  depends upon the total number of fingers and the ratio of gap-width to finger-width, and is of the nature shown in Fig. 10, and n is the order of the mode; and  $P(\phi)$  is a function depending upon the number and location of the phase-shifting fingers, as shown in Fig. 10.  $Z(\phi)$  can be analysed into its Fourier components as follows:

$$Z(\phi) = \frac{4}{\pi} \left\{ \sin \frac{\pi}{2\rho} \cos M\phi + \frac{1}{3} \sin \frac{3\pi}{2\rho} \cos 3M\phi + \frac{1}{5} \sin \frac{5\pi}{2\rho} \cos 5M\phi + \dots \right\},$$

where the ratio of gap width to finger width is  $1/(\rho-1)$ , and M is half the total number of fingers.

When no phase-shifting fingers are present, then for higher-order modes,

$$E_{\phi}(\mathbf{a}, \phi) = Z(\phi)\cos n\phi$$

$$= \frac{2}{\pi} \left[ \sin \frac{\pi}{Z\rho} \left\{ \cos(\mathbf{M} + \mathbf{n}) \phi + \cos(\mathbf{M} - \mathbf{n})\phi \right\} \right]$$

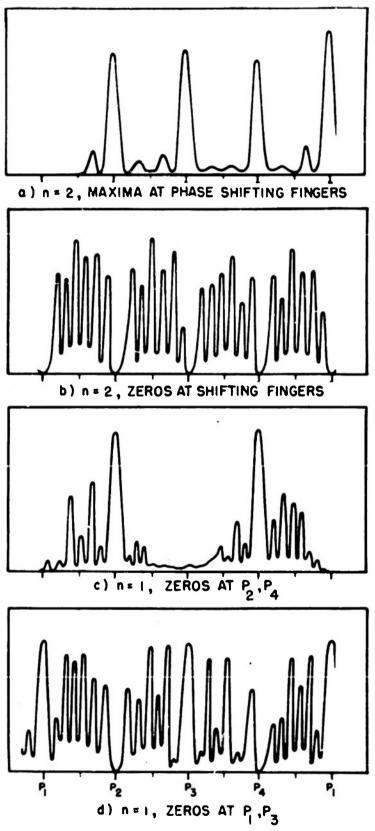
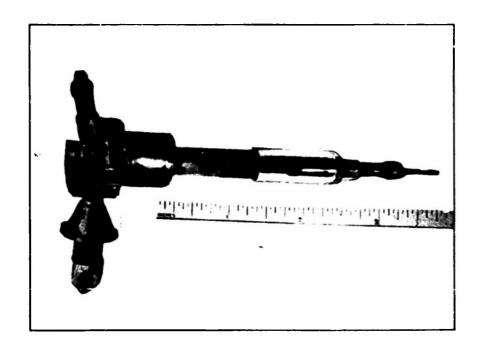


FIG. 7 FIELD PATTERNS WITH VANES



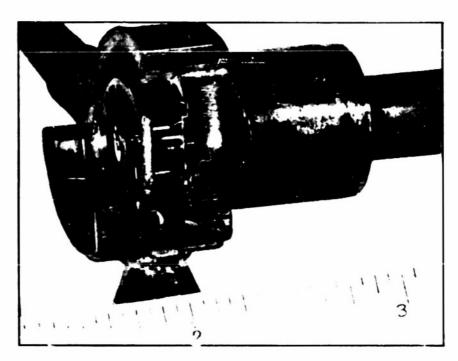
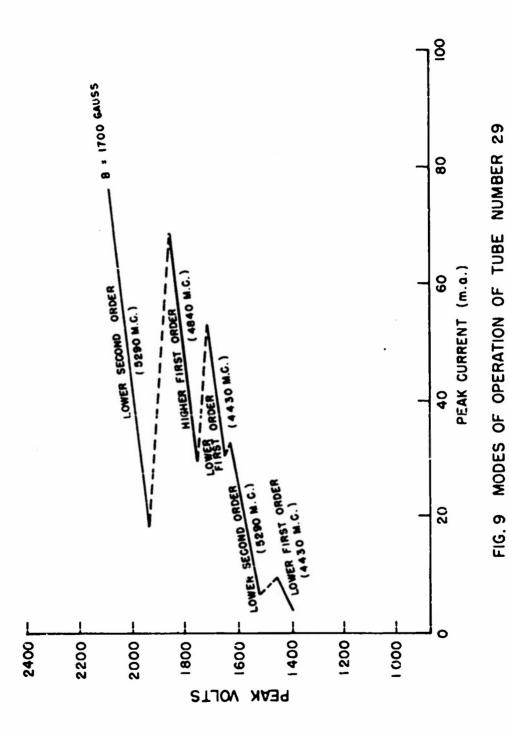


FIG. 8 A CUT-AWAY VIEW OF AN OPERATING TUBE



The state of the s

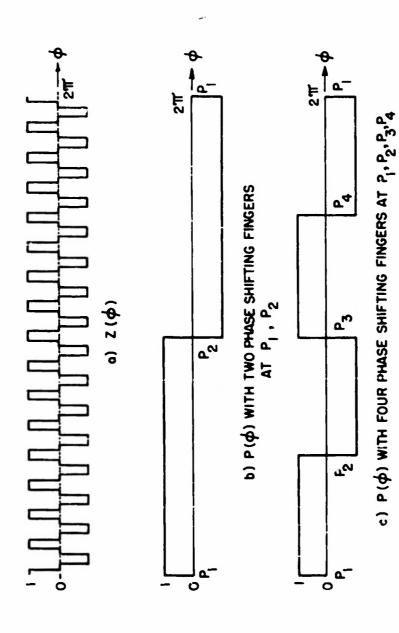


FIG.10 FUNCTIONS USED FOR OBTAINING FOURIER COMPONENTS OF FIELD CONFIGURATIONS

$$+\frac{1}{3}\sin\frac{3\pi}{2\rho}\left\{\cos(3M+n)\phi+\cos(3M-n)\phi\right\}+\ldots\right].$$

Alternatively

$$E_{\phi}(\mathbf{a},\phi) = Z(\phi)\sin n\phi$$

$$= \frac{2}{\pi} \left[ \sin \frac{\pi}{2\rho} \left\{ \sin(\mathbf{M} + \mathbf{n})\phi - \sin(\mathbf{M} - \mathbf{n})\phi \right\} + \frac{1}{3} \sin \frac{3\pi}{2\rho} \left\{ \sin(3\mathbf{M} + \mathbf{n})\phi - \sin(3\mathbf{M} - \mathbf{n})\phi \right\} + \dots \right].$$

Let  $\gamma$  represent the number of complete cycles around the anode, for a given Fourier component. It is seen that when no phase-shifting fingers are present,  $\gamma$  assumes the values:  $M\pm n$ ,  $3M\pm n$ ,  $5M\pm n$ , etc.

The case where four phase-shifting fingers are present is considered next. If in that case  $P(\phi)$  is denoted by  $P_{A}(\phi)$ , then by Fourier analysis,

$$\begin{split} P_{4}(\phi) &= \frac{4}{\pi} \sin 2\phi + \frac{4}{3\pi} \sin 6\phi + \frac{4}{5\pi} \sin 10\phi + \dots \\ P_{4}(\phi) \sin \phi &= \frac{2}{\pi} \cos \phi - \frac{2}{\pi} \cos 3\phi + \frac{2}{3\pi} \cos 5\phi - \frac{2}{3\pi} \cos 7\phi + \dots \\ P_{4}(\phi) \cos \phi &= \frac{2}{\pi} \sin \phi + \frac{2}{\pi} \sin 3\phi + \frac{2}{5\pi} \sin 5\phi + \frac{2}{3\pi} \sin 7\phi + \dots \\ P_{4}(\phi) \sin 2\phi &= \frac{2}{\pi} - \frac{4}{3\pi} \cos 4\phi - \frac{4}{15\pi} \cos 8\phi - \dots \\ P_{4}(\phi) \cos 2\phi &= \frac{8}{3\pi} \sin 4\phi + \frac{16}{15\pi} \sin 8\phi + \frac{24}{35\pi} \sin 2\phi + \dots \end{split}$$

The most significant Fourier components of  $E_{\phi}(a,\phi)$  would be obtained from the product of the first term in the expansion of  $Z(\phi)$  and the function  $\cos n\phi P_{\phi}(\phi)$  or  $\sin n\phi P_{\phi}(\phi)$  appropriate to the given mode. The components given by the subsequent terms in the  $Z(\phi)$  expansion would have their  $\gamma$  and their excitation voltages far removed from those corresponding to the first term. Taking unity as the coefficient of this first term, and expressing the products as sums and differences, one obtains:

$$\begin{split} P_4(\phi)\cos M\phi &= \frac{2}{\pi} \left\{ \sin(M+2)\phi - \sin(M-2)\phi \right\} + \frac{2}{3\pi} \left\{ \sin(M+6)\phi - \sin(M-6)\phi \right\} + \dots \\ P_4(\phi)\sin \phi \cos M\phi &= \frac{1}{\pi} \left\{ \cos(M+1)\phi + \cos(M-1)\phi \right\} - \frac{1}{\pi} \left\{ \cos(M+3)\phi + \cos(M-3)\phi \right\} \\ &+ \frac{1}{3\pi} \left\{ \cos(M+5)\phi + \cos(M-5)\phi \right\} - + \dots \end{split}$$

$$P_{4}(\phi)\cos\phi\cos\phi\phi = \frac{1}{\pi} \left\{ \sin(M+1)\phi - \sin(M-1)\phi \right\} + \frac{1}{\pi} \left\{ \sin(M+3)\phi - \sin(M-3)\phi \right\} + \frac{1}{3\pi} \left\{ \sin(M+5)\phi - \sin(M-5)\phi \right\} + \dots$$

$$P_{4}(\phi)\sin2\phi\cos\phi\phi = \frac{2}{\pi} \cos\phi\phi - \frac{2}{3\pi} \left\{ \cos(M+4)\phi + \cos(M-4)\phi \right\} - \frac{2}{15\pi} \left\{ \cos(M+8)\phi + \cos(M-8)\phi \right\} + \dots$$

$$P_{4}(\phi)\cos2\phi\cos\phi\phi = \frac{4}{3\pi} \left\{ \sin(M+4)\phi - \sin(M-4)\phi \right\} + \dots$$

$$+ \frac{8}{15\pi} \left\{ \sin(M+8)\phi - \sin(M-8)\phi \right\} + \dots$$

The values of  $\gamma$  thus obtained from the first term are M+2, M+6, etc. for the zeroth-order mode; M+1, M+3, M+5, etc., for the first-order modes; M, M+4, M+8, etc., for one second-order mode; and M+4, M+8, etc. for the other second-order mode.

The effect of the phase-shifting fingers has been to give  $\gamma$  the value M in one of the second-order modes. However, at the same time, in all the higher-order modes one pair of components has been replaced by a number of them, whose amplitude does not fall off so rapidly as to make their excitation improbable.

The foregoing study is helpful in explaining the observed voltages of operation. For a given mode and magnetic field, the voltage is inversely proportional to  $\gamma$ , to a first approximation. This approximation was considered adequate because there was uncertainty in choosing values of voltage from the data for comparison with theory, as each mode of excitation was obtained over a range of voltage. The starting point of each range was chosen for comparison with theory.

The following limitations of the above simple theory have to be recognized. The presence of any irregularities in the geometry of the resonator would introduce Fourier components not given by the above analysis. Examples of such irregularities are the coupling loop, as imperfectly aligned cathode, and irregular spacing of fingers. Apart from this, it is to be noticed that for Fourier analysis the range of azimuth over which the function  $E_{\delta}(a, \phi)$  was defined was 0 to  $2\pi$  However, in exciting a given

field configuration, the electrons need not be in synchronism with the field over this whole range. The following example illustrates the kind of difference between simple theory and experiment expected on the above basis. In the first-order mode, when four phase-shifting fingers are present, a value of  $\gamma = M$  is not given by Fourier analysis. However, an excitation voltage corresponding to  $\gamma = M$  would be possible, if an electron, while giving energy to the r-f field, could-reach the anode without crossing the two phase-shifting fingers, which lie at the maxima of the E-field.

## Explanation of Experimental Results

In these tubes M was equal to 16, as the tube had 24 ordinary fingers and four phase-shifting fingers, each one of the latter being equivalent to two ordinary ones. Data were taken on the two first-order modes, and the second-order mode whose nodal planes were at the phase-shifting fingers. The zeroth-order mode could not be operated since cathode decoupling chokes were not used. Operation in the other second-order mode was erratic, since it had eight points of phase reversal around the anode.

Table II

Tube Number	Mode	Magnetic Field in Gauss	Proportions of Reciprocals of Voltages
22	Lower Second	1 1370	16.0:17.7:20.2
22	Lower Second	1920	16,0:18.3:19.9
23	Higher First	1920	16.0:17.2
23	Lower Second	1 1920	16.0:18.0
28	Lower Second	1700	16.0:19.9:20.8:22.0
28	Lower Second	2260	16, 0:17, 3:20, 1:21, 3
29	Lower First	1480	13.3:16.0:19.2
29	Lower First	1700	16.0:19.1:21.3
29	Lower First	2140	16.0:19.1
29	Lower First	2610	16.0:19.0
29	Lower Second	1480	16.0:20.5
29	Lower Second	1700	16, 0 : 20, 0
29	Lower Second	2140	16.0:20.5
29	Lower Second	2610	16.0:20.5

The data given in Table II may be discussed under the following headings:

- a. Within experimental error, the reciprocals of voltages for each mode are proportional to numbers in the series, 16, 16±1, 16±2, etc. Table II shows the actual numbers obtained in the series, 16 being taken as the reference number.
- b. The presence of voltages corresponding to  $\gamma = 16$  and 20 in the second-order mode and to  $\gamma = 13$ , 17, 19 and 21 in the first-order modes is in conformity with values obtained from Fourier analysis.
- c. The presence of a voltage corresponding to  $\gamma = 16$  in the first-order modes is due to the fact that electrons can reach the anode by covering only a small range in azimuth, as already discussed. The assumption that the voltage corresponds to  $\gamma = 16$ , was checked by using the voltage for  $\gamma = 16$  in the second-order mode as reference. The voltages were found to be directly proportional to the frequencies to within 1 per cent, as would be expected when  $\gamma$  is the same for the two cases.
- d. The occasional presence of voltages corresponding to other values of γ may be ascribed to irregularities in the structure, which would modify the field and so introduce additional Fourier components.
- e. The consistent absence of a voltage corresponding to  $\gamma$  = 17 from the lower first-order mode is understandable, since the same order of voltage can excite the  $\gamma$  = 20 component of the lower second-order mode; and the latter appears to be more easily excited.

### IV. Conclusions

By proper choice of the ratio of cavity radius to anode radius of an interdigital resonator, a basic separation of 40 per cent or more can be obtained
between modes of various orders. Operation in only one mode cannot be ensured by short-circuiting those fingers to the opposite face of the cavity at which
the E-field of the desired mode vanishes. The frequencies of other modes
increase, but the modes may still be operable. The frequency spectrum can
become quite confusing as a result of this. Sensitive control over the various
modes can be obtained by introducing radial vanes in the cavity. By proper
choice of the radial penetration of the vanes not only can the degenerate pairs
of modes be separated out, but also all the modes can be accurately located
at equal intervals.

The field configurations of these modes resemble that of the  $\pi$ -mode in a multicavity magnetron. However, in the higher-order modes, since an additional cyclic variation with  $\phi$  is superimposed over the  $\pi$ -mode configuration, the Fourier components occur as nearby pairs. This would give rise to two nearby voltages of operation for each mode, only one pair of components being of importance. Phase-shifting fingers do not ensure that operation would be obtained at only one voltage for one mode, or that the other modes will become inoperative. In fact, with phase-shifting fingers, several components, close to one another, are obtained instead of a pair of them, for each mode. Thus, phase-shifting fingers are of doubtful advantage.

Before the advantage of a number of well-separated, easily controlled modes can be exploited, the problem of voltages of operation for the higher-order modes will have to be studied further. One approach would be to increase the number of fingers, without using phase-shifting fingers. The percentage separation of the two components would thus be reduced. Considering the fact that operation at a voltage corresponding to  $\gamma = M$  is also possible, in addition to that in which  $\gamma = M + n$ , it appears possible that all three ranges may merge into one continuous range, especially for n = 1. In this way, at least three modes of operation may be obtained, in three distinct ranges of voltage; namely, the zeroth-order and the two first-order modes.

## Acknowledgements

I thankfully acknowledge the guidance and personal encouragement I received from Professor E. L. Chaffee, Dr. D. L. Benedict and Professor R. W. P. King. The work of Dr. C. Yeh and Mr. J. P. Jasionis on operating tubes using shorting wires provided the stimulus for further work on mode control. The observation that the interdigital magnetron operated at more than one voltage for a given mode of resonance was also first made by them. The data on operating voltages of tubes number 22 and 23 were taken by Dr. Yeh.

## References

- 1. Crawford and Hare, "A Tunable Squirrel-Cage Magnetron, the Donutron," Proc. I.R.E. 35, 361-369 (1947)
- 2. Hull and Randalls, "High-Power Interdigital Magnetrons," Proc. I.R.E. 36, pp. 1357-1363 (1948).